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Acoustic band gaps for a two-dimensional periodic array of solid cylinders in viscous liquid

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Abstract
We investigated the absolute acoustic band gaps for two-dimensional periodic arrays of silica cylinders in viscous liquid. Acoustic band structures, which have complex eigenfrequencies for these systems, are calculated with the plane wave expansion method. They show that when the viscous penetration depth is comparable to the structural length scale, the structures possess large absolute acoustic band gaps compared with those without viscosity. We also found that the magnitude of the viscosity plays an important role in these band gaps.

1. Introduction

In the past decade, significant effort has been devoted to the study of photonic crystals because of their novel physical properties and many potential applications [1, 2]. Recently, a great deal of attention has extended to the phononic crystals, the counterpart of photonic crystals, for which elastic (EL) waves and/or acoustic (AC) waves are concerned [3–25]. As in the photonic crystals, the basis of all applications of phononic crystals, such as acoustic/elastic wave filters, depends on the existence of wide frequency band gaps in which no sound and vibration are allowed. In addition, because of elastic waves' vector characters and the possible coupling of their longitudinal and transverse modes, rich physics is expected to exist in EL and AC waves propagating in phononic crystals.

Since Mártinez-Sala *et al* found that sound can be attenuated by the two-dimensional (2D) array of rigid cylinders periodically arranged in air [9], the AC or EL waves propagating in solid–liquid, liquid–liquid and solid–solid systems have been studied extensively [10–25]. The formation of an acoustic band gap in binary composite strongly depends on the lattice structure [13, 14] and the material properties of each component, such as mass density, wave velocities or elastic moduli [3–8]. Because of the suppression of transverse sound in the liquid, the acoustic gaps are comparatively difficult to produce in solid–liquid systems. However, it

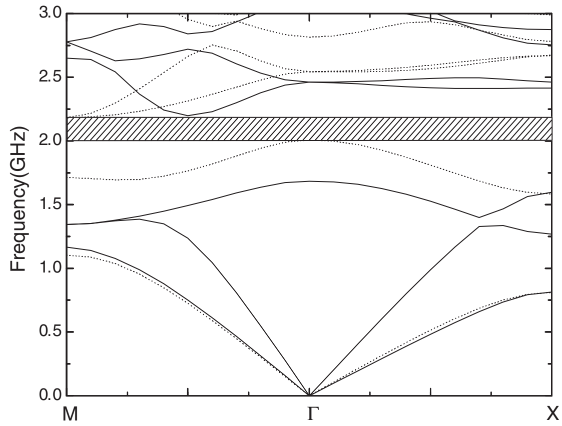

Figure 1. The acoustic band structure of silica cylinders arranged in a square lattice in an ice matrix. The lattice constant $a = 1.0$ μm and the filling fraction of silica cylinders is 0.4. The frequency gap range is marked by the shaded area.

was found recently that the viscosity in the liquid can help to open acoustic band gaps in three-dimensional (3D) systems [10]. The shear viscosity in a liquid brings a new length scale associated with the penetration of shear stress into the viscous liquid. If the viscous penetration depth δ ($\delta = (2\eta/\rho\omega)^{1/2}$, where η is the shear viscosity, ρ is the density of the fluid and ω is the angular frequency) is comparable to the structural length scale of the composite, viscous effects should not be ignored for the acoustical properties.

In this paper, we investigate the effect of the viscosity on the 2D band structures of the periodic arrays of solid cylinders in viscous liquid. We not only found the similar effect as in 3D systems, namely that there exists gap formation in the solid–liquid system [10], but also found that when the viscous penetration depth is comparable to the structural length scale (the composite periodicity), the 2D solid–viscous fluid materials possess larger absolute acoustic band gaps than those without viscosity. In particular, the dependence of the gap size on the magnitude of the viscous damping parameter is also investigated in this paper.

For the 2D structure consisting of the infinite cylinders parallel to the z axis and for the wave propagating in the x, y plane normal to the z axis, the displacement can be described by two independent wave equations. One describes a pure transverse mode polarized in the z direction displacement, which may be expressed as

$$-\omega^2 u^z = \frac{1}{\rho(\mathbf{r})}\nabla(\mu(\mathbf{r})\nabla u^z),$$

and the other describes the mixture of longitudinal and transverse modes polarized in the x, y plane, which may be expressed as

$$\frac{\partial^2 u^i}{\partial t^2} = \frac{1}{\rho(\mathbf{r})}\left\{\frac{\partial}{\partial x_i}\left(\lambda(\mathbf{r})\frac{\partial u^1}{\partial x_1}\right) + \frac{\partial}{\partial x_1}\left[\mu(\mathbf{r})\left(\frac{\partial u^i}{\partial x_1} + \frac{\partial u^1}{\partial x_i}\right)\right]\right\},$$

where $\rho(\mathbf{r})$ is the density and $\lambda(\mathbf{r})$, $\mu(\mathbf{r})$ are the Lamé constants of the medium. We call the former mode a single mode and the latter two modes coupled to each other a mixed mode. For the solid–pure liquid system, there is only a longitudinal mode in the fluid. For the solid–viscosity liquid system, there exists shear viscosity η in a fluid which means complex elastic constants $\mu = -\omega\eta\mathrm{i}$ and $\lambda = \rho C^2 + (2/3)\omega\eta\mathrm{i}$ [10] for the liquid. Correspondingly, the complex sound velocities in the liquid are $c_1^2 = (\rho C^2 - (4/3)\omega\eta\mathrm{i})/\rho$, $c_t^2 = -\omega\eta\mathrm{i}/\rho$. In order to avoid a

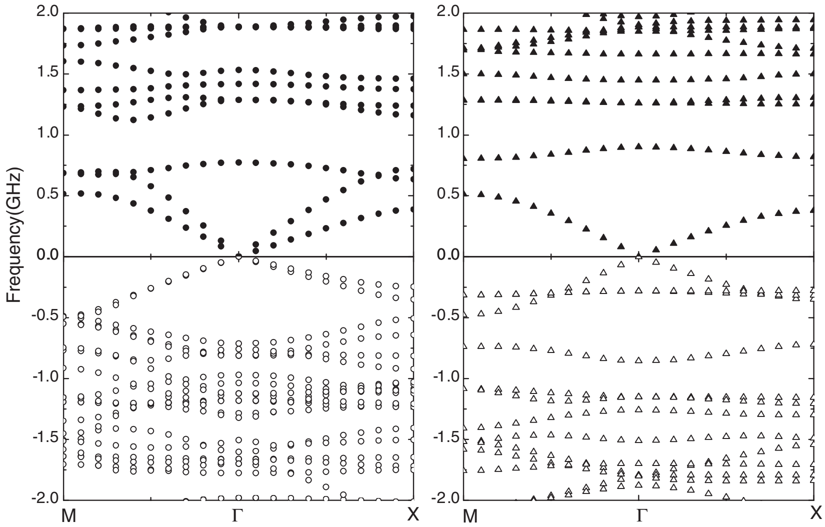

Figure 2. The acoustic band structure of silica cylinders arranged in a square lattice in a liquid with viscous damping parameter $\theta = 1.2 \times 10^9$. The lattice constant $a = 1.0$ μm and the filling fraction of silica cylinders is 0.4. The complex eigenfrequencies are plotted for the mixed mode in the left-hand panel and for the single mode in the right-hand panel. The real frequency ω' is given in the upper part and the imaginary frequency ω'' representing the damping in the lower part.

complicated diagonalization problem originating from the explicit frequency dependence, we introduce a viscous damping parameter $\theta = \omega\eta$ and assume it is frequency independent, as dealt with in the 3D case [10] (this assumption will not introduce much discrepancy in the short frequency region for the band gap we are interested in). Then, the complex elastic constants can be written as $\mu = -\theta \mathrm{i}$ and $\lambda = \rho C^2 + (2/3)\theta \mathrm{i}$. The calculations of band structure are performed with the popular plane-wave expansion (PWE) method. 441 plane waves are used in the expansion and the numerical results reach a good convergence.

2. Results and discussions

As a characteristic example to show the importance of the role of transverse mode in the system, we first calculated the band structure for a solid–solid system consisting of silica cylinders embedded in an ice matrix. The material parameters used in the calculations are $\rho = 2.2$ kg m^{-3}, $C_l = 5.97$ km s^{-1}, $C_t = 3.76$ km s^{-1} for silica, and $\rho = 0.94$ kg m^{-3}, $C_l = 3.83$ km s^{-1}, $C_t = 1.84$ km s^{-1} for ice, where ρ, C_l and C_t are, respectively, the density, and the longitudinal and transverse sound velocity. Figure 1 shows the band structure of a square lattice of silica cylinders with a volume fraction $f = 0.4$ in ice matrix. The diameter of the cylinders is 0.714 μm and the lattice constant $a = 1.0$ μm. In the figure, the solid curves describe the disperse relation of the mixed mode and the dashed curves stand for the single mode. Near 2.1 GHz, it exhibits a small band gap between the fifth and the sixth bands with width 0.176.

We now focus on the AC wave propagating in a viscous solid–liquid system including silica cylinders in a viscous liquid. As we expected, when the viscous length δ is comparable to the structural length scale of the composite, the viscosity has an important effect on the acoustical properties. In the 2D band structure calculations, the parameters of the liquid are chosen as

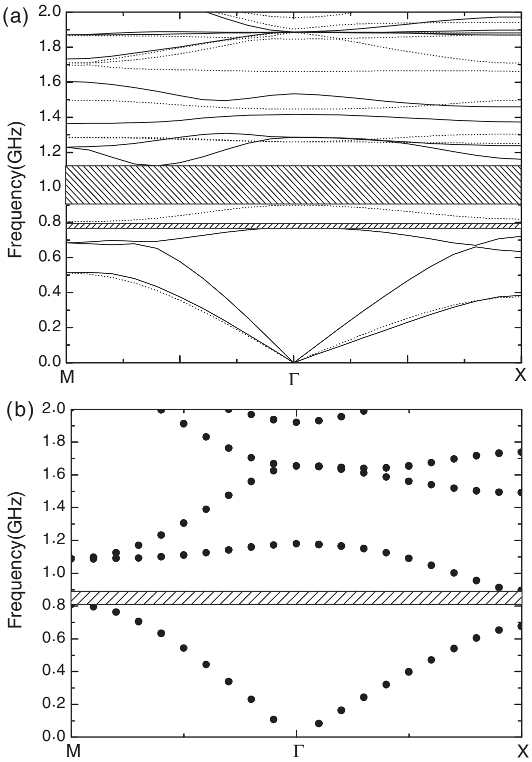

Figure 3. (a) The gap formations of silica cylinders arranged in a square lattice in a liquid with viscous damping parameter $\theta = 1.2 \times 10^9$. (b) The AC wave band structure of silica cylinders in pure liquid. The lattice constant $a = 1.0\ \mu$m and the filling fraction of silica cylinders is 0.4.

$\rho = 1.0$ kg m^{-3}, $C_l = 1.48$ km s^{-1}, and complex elastic constants μ and λ with viscous damping parameter $\theta = 1.2 \times 10^9$ (this corresponds to a shear viscosity $\eta = 1.2/(2\pi)$ Pa s for the liquid at frequency 1 GHz, and a penetration depth of order of μm) are used for the liquid. The complex eigenvalues $\omega = \omega' + \omega''$i are obtained by solving the dynamical equation with complex elastic constants and real wavevectors. The imaginary part ω'' gives the damping of the mode with time. The numerical result of the dispersion relation with complex frequency is shown in figure 2. From it, we can also see the mixed mode (left-hand panel) and the single mode (right-hand panel) just as in 2D solid–solid systems. Because of the viscous effects, a transverse mode can exist in the viscous liquid. One transverse mode coupled with the longitudinal mode polarizes in the x, y plane and the other transverse mode polarizes in the z direction. It is also found that the mixed mode and single mode are all highly damped, which is demonstrated by the imaginary part of the frequency shown in the bottom of the picture.

At the same time, near 0.78 and 1.01 GHz one observes the gap formations. Figure 3(a) shows the first band gap between the forth and fifth bands and the second gap between the fifth and sixth bands. The width of the first gap is 0.032 and that of the second is 0.223. Comparing the result with that of the silica cylinders in the liquid ignoring the viscosity (see figure 3(b); this is calculated with the multiple-scattering method [25], applicable for calculating the disperse relation of the solid–liquid system), there is only a smaller gap with width 0.087 near the 0.85 GHz. This implies that introducing the viscosity in the liquid can produce a large band gap. It should be pointed out that the presence of absorption in the solid–viscous fluid system

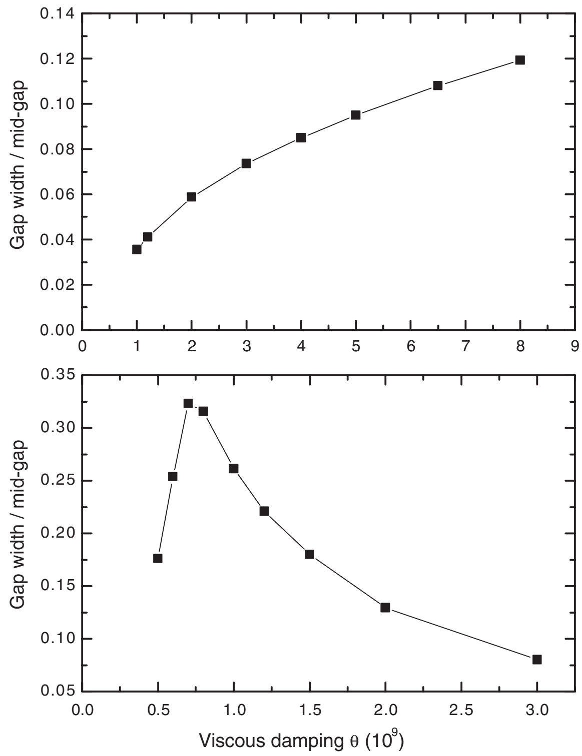

Figure 4. Gap width/mid-gap frequency versus the viscous damping parameter θ. The case for the first gap is shown in the upper part and that for the second gap is shown in the lower part.

tends to blur the distinction between pass bands and stop bands, but the band gap for an absorptive system is still capable of distinguishing from a combined measurement for both the transmission and reflection coefficients in an experiment.

It has been seen that the viscous damping in the liquid results in the larger gap formation in solid–liquid systems. Furthermore, we will investigate the effect of the various viscous damping parameters on the gap properties. Figure 4 shows the gap width/mid-gap frequency versus the viscous damping parameter θ. The upper part presents the behaviour of the first gap shown in figure 3 and the lower part presents that for the second gap. From the figure, we can see that the first gap width increases gradually when the viscous damping parameter θ runs from 1.0×10^9 to 8.0×10^9. For the second gap, there is a peak at $\theta = 0.7 \times 10^9$ (the shear viscosity $\eta = 0.7/(2\pi)$) and the corresponding maximum of the gap width/mid-gap is 0.323. When the viscous damping is larger than 0.7×10^9, the gap width decreases with the viscous damping increasing.

3. Conclusions

In conclusion, by using the PWE method, we studied the absolute acoustic band gaps in 2D systems consisting of silica cylinders in a viscous liquid. It was shown that if the viscous penetration depth is comparable to the structural length scale, viscous effects play an important role in the acoustical properties. We calculated the acoustic band structure with complex eigenfrequency for the viscous parameter $\theta = 1.2 \times 10^9$, which corresponds to a viscosity

$\eta = 1.2/(2\pi)$ at frequency 1 GHz in the liquid. Numerical results showed that the 2D Si–viscous liquid composite exhibits large absolute AC wave band gaps compared with the Si–pure liquid system. These gaps are also found to depend on the choice of the viscosity in the liquid.

Acknowledgments

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